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GEODETIC ANALYSIS OF SKYLAB ALTIMETRY PRELIMINARY DATA-SL/2 EREP PASS 9

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ABSTRACT

GEODETIC ANALYSIS OF SKYLAB ALTIMETRY PRELIMINARY DATA

The analysis was based on a time series intrinsic relationship between the satellite ephemeris, altimeter measured ranges, and the corresponding a priori values of subsatellite geoidal heights. Using, sequential least squares processing with parameter weighting, the objective was to recover (1) the absolute geoidal heights of the subsatellite points, and (2) the associated altimeter calibration constant(s). Preliminary results from Skylab Altimetry are given, using various combinations of orbit ephemeris and altimeter ranges as computed differently by NASA/JSC and NASA/Wallops. The influences of orbit accuracy, weighting functions and a priori ground truth are described, based on the various combination solutions. It is shown that to deduce geoidal height by merely subtracting the height of the satellite from the altimeter range is inadmissible.

In particular, the results of such direct subtraction can be very misleading if the orbit used is computed from data that included altimeter data used as height constraints. In view of the current state of our knowledge of (1) satellite altimeter biases and (2) radial errors in orbit computation relative to geocenter, and because satellite altimetry is a "geodetic geometric leveling from space", the use of geodetic ground truth samples as control "benchmarks" appears indispensable for the recovery of absolute geoidal heights with correct scale. Such geodetic ground truth in the oceans have to be determined from marine geodetic techniques involving astrogravimetry and satellite geodesy.

ACKNOWLEDGEMENT

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GEODETIC ANALYSIS OF SKYLAB ALTIMETRY PRELIMINARY DATA - SL/2 EREP PASS 9

INTRODUCTION

The "Williamstown Study" [Kaula, 1970] recommendation for the use of spacecraft altimeters for geodetic, geophysical and oceanographic studies of the oceans and the earth's gravity field was implemented for the first time in history under Skylab's experiment S-193. Battelle's Columbus Laboratories was awarded the only contract for "Calibration and Evaluation of Skylab Altimetry for Geodetic Determination of the Geoid". The S-193 altimeter experiment is one of a number classified under "Earth Resources Experiments Package" (EREP) whose end objectives are to solve various problems on earth, that directly affect even the man in the street.

Three manned Skylab missions--SL/2, SL/3, and SL/4, are to provide data from the S-193 system. Geodetic analysis of Skylab S-193 altimeter preliminary data from mission SL/2 and EREP pass number 9 is the subject of this paper. The overall objective of the investigation is to demonstrate the feasibility of and necessary conditions for determination of the Marine Geoid (i.e., the geoid in ocean areas) from satellite altimetry. The geoid is the equipotential surface that would coincide with "undisturbed" mean sea level of the earth's gravity field. "Undisturbed" is the condition that would exist if the oceans were acted on by the force of gravity only and no other forces such as due to ocean currents, winds, tides, etc. Thus, determination of the geoid (mean sea level - msl) is basic to understanding of the oceans and their dynamic phenomena such as currents, tides, circulation patterns and hence air-sea interactions. Improved numerical weather predictions require accurate knowledge of these ocean dynamics phenomena. Navigation, waste disposal and pollution control also benefit from an accurate knowledge of ocean dynamics. More accurate determination of the geoid will lead to a better definition of the earth's gravity model. Computation of the global geoid by conventional methods is so expensive and time consuming and are beset with so many problems as discussed in Fubara and Mourad [1972a] that these conventional techniques cannot be depended on for completion of the job in the forseeable future. These factors justify the need for new systems, and techniques and current indications are that satellite altimetry may be the answer.

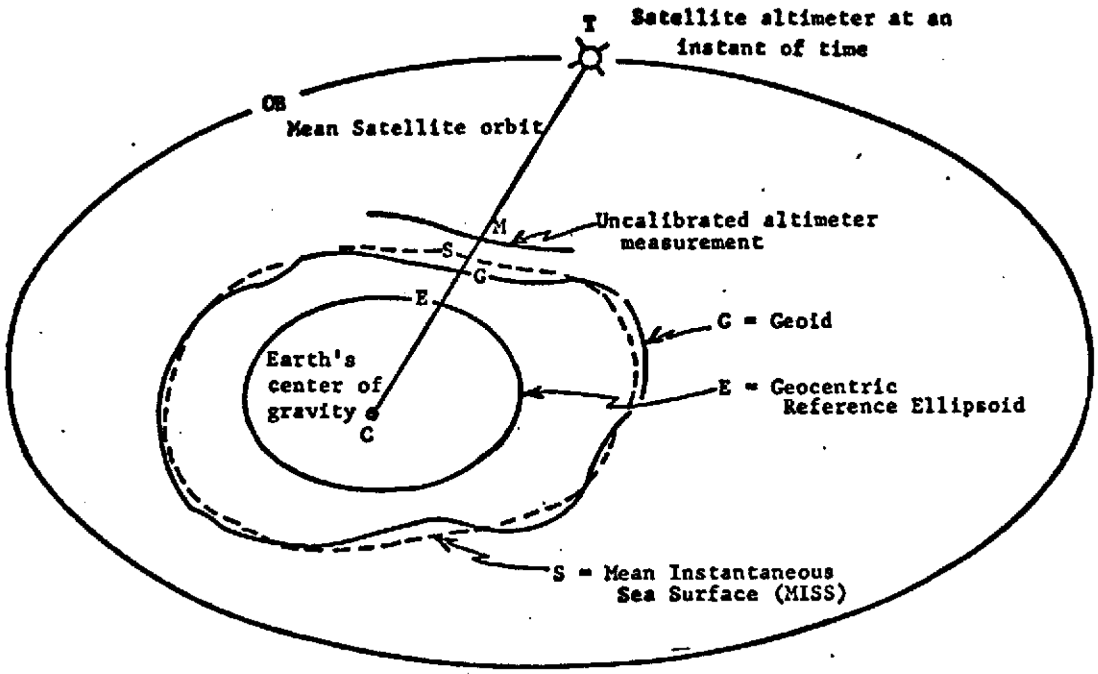

FIGURE 1. SCHEMATIC GEOCENTRIC RELATIONS OF SURFACES INVOLVED IN SATELLITE ALTIMETRY

Figure 1 shows schematic geocentric relations of the various surfaces associated with satellite altimetry. TM is the raw altimeter range which has to be corrected for laboratory instrumental calibration, electromagnetic effects, sea state, and periodic sea surface influences to give TS. S represents the non-periodic "sea level". CT and CE, the geocentric radii of the altimeter and E, its subsatellite point on the reference ellipsoid, are computed from satellite tracking information. EG is the absolute geoidal undulation to be computed from this investigation.

ANALYTICAL DATA HANDLING FORMULATIONS

Condition Equation of Intrinsic Parameters

Each measured altimeter range R_i^o with an associated measurement residual v_i is intrinsically related to (1) X_s, Y_s and Z_s (the satellite coordinates at the instant of measurement), (2) the geoidal undulation N_i^a (of the subsatellite point) based on a reference ellipsoid of parameters a, and e, and (3) the biases in all measurement systems involved. The condition equation for this intrinsic relationship can be stated as:

$$v_i + R_i^o(1 + \Delta f) - D_i \pm N_i^o + \Delta N_i = 0 \tag{1}$$

where

$\Delta f = f_1$(systematic errors in X_s, Y_s, Z_s, the altimeter bias and sea state correction bias) is the total system calibration constant to be determined,

$N_i^a = N_i^o + \Delta N_i$ (N_i^o is an approximate value for N_i^a)

and

$D_i = f_2\ (X_s, Y_s, Z_s, a, e)$.

The exact functional mathematical expression for Δf is unknown and is treated later.

D_i is essentially the geodetic height of the satellite above the chosen reference ellipsoid and is given by

$$D_i = (X_s^2 + Y_s^2)^{1/2}\ \mathrm{Sec}\ \varphi - a(1-e^2\mathrm{Sin}^2\varphi)^{-1/2} \tag{2a}$$

or

$$D_i = Z_s\mathrm{Cosec}\ \varphi - a(1-e^2\mathrm{Sin}^2\varphi)^{-1/2}(1+e^2) \tag{2b}$$

However, usually φ in Equation (2) is not known and has to be derived from

$$\varphi_i = \tan^{-1}\left[\frac{Z_s + e^2a(1-e^2\mathrm{Sin}^2\varphi)^{-1/2}}{(X_s^2 + Y_s^2)^{1/2}}\right] \tag{3}$$

Equation (3) is usually not solved directly except as recently developed by Paul [1973]. Solving for D_i and φ_i from the given X_s, Y_s and Z_s was done iteratively. By putting $D_i = 0$, the first approximation for φ_i is

$$\varphi \cong \tan^{-1}\left[Z_s(X_s^2 + Y_s^2)^{-1/2}(1-e^2)^{-1/2}\right] \tag{4}$$

This φ is then used in Equation (3) which is iteratively solved from $i = 1, \ldots n$ until

$\varphi_n - \varphi_{n-1} \leq \Delta\varphi$ which is usually set at $\Delta\varphi = 0.001$ arc second. Thereafter, D_i is computed from Equation (2a) or (2b).

Generalized Least Squares Adjustment Model

Equation (1) can be rewritten in matrix form as

$$F_1\ (X_1^a, X_2^a, L_1^a) = 0, \tag{5}$$

subject to the normalized weighting functions P_1, P_2 and P_3 associated with X_1, X_2 and L_1, respectively. Relating Equations (1) and (5) explicitly,

$$X_1^a = N_i^o + \Delta N_i \tag{6}$$

$$X_2^a = \Delta f \tag{7}$$

$$L_1^a = R_i^o + V_i \tag{8}$$

In this model, all parameters and measurements of the mathematical model are treated as "measurements" and weighted accordingly. Thus, constants (fixed variables) have infinitely large weights ($P = \infty$) because they need no corrections (residuals) and as residuals tend towards zero, the corresponding weight approaches infinity. Unknown parameters (free variables) in the classical sense have weights $P = 0$. All other "measurements" have finite weights $0 < P < \infty$. This mathematical model for the generalized least squares processing of experimental data is based on works of Schmid and Schmid [1964], Fubara [1969 and 1973].

The superscript "a" denotes the exact true values of the "measurements". Usually, these true values are not known. Instead, the corresponding measured or approximate values X_1^o, X_2^o, and L_1^o with associated variance-covariances P_1^{-1}, P_2^{-1}, P_3^{-1}, are estimated or measured. Therefore, Equation (5) can be rewritten in the form

$$F_2\left[(X_1^o + \Delta_1),\ (X_2^o + \Delta_2)\ ,\ (L_1^o + V_1)\right] = 0 \tag{9}$$

where

$$X_1^a = X_1^o + \Delta_1$$

$$X_2^a = X_2^o + \Delta_2$$

$$L_1^a = L_1^o + V_1$$

The linearized form of Equation (9) is

$$A_1\Delta_1 + B_1\Delta_2 + C_1V_1 + F_2\ (X_1^o,\ X_2^o,\ L_1^o) = 0 \tag{10}$$

A_1, B_1, and C_1 are the first partial derivatives in a Taylor series expansion of Equation (9), associated with X_1^o, X_2^o, and L_1^o, respectively, while Δ_1, Δ_2, and V_1 are the correction parameters to be determined. The least squares solution of Equation (10) to derive the corrections Δ_1, Δ_2, and V_1 to "measured" X_1^o, X_2^o and L_1^o is as developed in Fubara [1973].

PRELIMINARY DATA ANALYSIS AND EVALUATION

The analytical data handling formulations for this investigation call for the following basic inputs: (1) the altimeter ranges, and exact time (usually GMT) of each measurement to correlate it with (2) the associated orbit ephemeris, and (3) geoidal information used as geodetic control or benchmark along the subsatellite track to help define the geodetic scale of the outputs. The main outputs are: (1) the residual bias of the altimeter or calibration constant required to give a correct absolute geoidal scale, and (2) the geoidal profile, both deduced from the computer processing of the imputs using least squares processing with parameter weighting according to the aforementioned formulations.

Tabulated data from mission SL/2, EREP pass #9, used in this paper, were obtained from NASA/JSC. We also obtained from NASA/Wallops the orbit ephemeris and altimeter ranges they computed independently for EREP pass #9. The NASA/JSC data differ significantly from the NASA/Wallops data, mainly in terms of scale and their computed geographic locations. Preliminary evaluation of the data indicate that in general they are good for processing. Apart from the scale problem, the altimeter ranges look much more consistent than had been anticipated. There are, however, some irregularities in the data received. These are being resolved. The independently computed altimeter ranges and orbit ephemeris received from NASA/JSC and NASA/Wallops present four different data combinations that were processed. These various combination solutions were used in the analyses of (1) the efficiency of the data handling formulations, (2) the influences of orbit errors, and (3) the role of the choice of a priori geoidal ground truth. Some schools of thought believe that geoidal heights could be obtained by merely subtracting the geodetic heights of the satellite from the corresponding altimeter ranges. We computed and evaluated results from such a method which we consider invalid due to certain physical limitations of the data.

RESULTS AND ANALYSIS

From the given satellite orbit and measured altimeter ranges, the overall objective of the investigation is to simultaneously (a) determine a geodetic calibration constant(s) that (b) corrects or adjusts the altimeter ranges for (c) determination of absolute geoidal heights with correct scale. Tables 1 to 3 and Figures 2 and 3 show the geodetic heights of the orbits and the altimeter ranges as computed by NASA/JSC and NASA/Wallops. (All Tables and Figures are at the end of the text.)

Calibration Constants and Adjusted Altimeter Ranges

As developed earlier, the altimeter bias, radial errors in orbit determination, and errors from inadequate or total lack of correction for significant sea state variations are all algebraically additive. These errors are inseparable unless two of them are absolutely known. In this investigation, the total sum of all three is the calibration constant to be determined.

Unfortunately, unless the radial orbit error is zero, some known absolute geoidal height must be used as geodetic control or benchmark in order to determine the required calibration constant. In this case, the calibration constant so determined is scalewise-dependent on the a priori geoidal input or the geodetic control used. This is demonstrated in the graphs A, B, C, and D of Figure 4. In graph A, the input is zero for a priori (approximate) geoid heights and no point is held geodetically fixed relative to another. For graph C, instead of zero, the a priori geoidal height input was -45 meters for every point. In graphs B and D the approximate geoidal height inputs were taken from the geoid of Vincent and Marsh [1973], as shown in Figure 5. In graph D, no points were constrained but, in B, the first point (left end) was constrained by weighting. For each case, normalized parameter weighting, consistent with the estimated absolute accuracy of the a priori geoidal height input, was applied. In all cases, even though the resultant geoidal height differences were exactly identical, the deduced calibration constants depended on the weighted a priori geoidal height inputs. Figure 4 definitely shows that such a priori inputs affect only the linear scale of the calibration constant and not the shape of the deduced geoid. Further investigations on the role of the values and errors that were intentionally introduced into the a priori geoidal inputs and the results shown in Table 4 and Figure 6 confirm the above conclusion.

In the current Skylab data, the altimeter bias appears to vary with the modes and the sub-modes which are described in Kern and Katucki [1973]. This was another factor taken into account. For the current data processing, the additional assumption is that for a "short time interval", the systematic radial orbit errors are of constant magnitude and sign. These two factors constrain the current "short time interval" for this set of data to be no more than 3 minutes.

A key indicator of the reliability of the analytically computed geodetic calibration constant is the consistency of the adjusted ranges. There are currently some avoidable errors in the computed orbit-Wollenhaupt and Schiesser [1973]. The differences in both orbit and the range data as computed by NASA/JSC and NASA/Wallops (see Figures 2 and 3 and Tables 1 to 3) confirm that the knowledge of (1) the orbit and (2) the delay constants (biases) for transforming the radar altimeter returns into ranges in engineering units is inaccurate. The mathematical model developed for this analysis anticipated these problems which algebraically add up to be a linear radial error relative to the earth's geocenter. Through the use of the discussed appropriately weighted a priori geoidal heights; (a) no matter what the errors in the different sets of ranges used, the derived adjusted ranges should be identical if the same orbit is used; (b) alternatively, if a unique set of ranges is used with different orbit data, the adjusted set of ranges should differ by only the radial differences between the orbits. The expectations (a) and (b) are established to within the noise level of the data by the results of Tables 2 and 3.

Geoidal Heights Analytically Deduced from Satellite Altimetry

Table 5 and Figure 5 show the deduced geoidal heights from the analytical processing of the four data combinations already described. Figure 5 also shows three other profiles for the same segment of the geoid as given by Vincent, et al [1972 and 1973] using different conventional techniques. Our results do not match these other conventional geoid profiles which also disagree with each other significantly. These three are tilted relative to each other and to the general slope of the altimeter geoid. However, the overall slope of the altimeter geoid more closely identified with the slope of the conventional satellite geoid. The other two conventional geoid segments are primarily based on global gravity data and satellite-derived geopotential coefficients used in global areas lacking measured gravity data.

It is logical to assume that whatever systematic radial errors exist in the computed orbits for the short time period involved, such errors should be constant in magnitude and sign. It is therefore valid to assume that, provided the altimeter system is stable, the deduced altimeter geoid should more closely approximate the true geoid shape of that segment. So far both the influences of sea state and the departure of sea surface topography from the true geoid have been neglected.

By merely subtracting the measured altimeter ranges from the corresponding satellite geodetic heights, the resultant profiles for the four data combinations are shown in Table 6 and Figure 7. Some schools of thought believe that this is all there is to geoid computation from satellite altimetry. The results in Table 6 and Figure 7 show 4 surface profiles which, if assumed to be the same segment of the geoid, represent geoid heights in the range of (1) -42 to 48 meters, (2) -62 to -66 meters, (3) -135 to -139 meters and (4) -155 to -157 meters. In contrast, our preliminary analytically deduced corresponding 4 profiles represent geoid heights within -49 to -45 meters, -48 to -46 meters, -49 to -45 meters and -48 to -47 meters from the 4 data combinations as in Figure 5.

CONCLUSIONS

The preliminary conclusions from these quick-look data investigations and previous simulation studies include:

(1) The analytical data handling formulations developed for this investigation appear to be very satisfactory. The main outputs required, the geodetic calibration constant, the geoid height and the corrected altimeter ranges were reliably determined;

(2) To ensure that the deduced calibration constant and geodetic heights are absolute, the use of geodetic control or a benchmark whose absolute geodetic undulation is known is indispensable. The establishment of such controls from a combination of astrogravimetry and satellite data is discussed in Mourad and Fubara [1972], and in Fubara and Mourad [1972a] and the practical implementation is partially demonstrated in Fubara and Mourad [1972b]. There is an implicit correlation between this conclusion and the conclusion based on a different type of investigation in Rapp [1971] that: "In carrying out simulation studies with non-global data it was concluded that altimetry data could not be used alone for potential coefficient determination.... Consequently, the altimetry data was combined with geoid undulation information in non-ocean blocks and with existing terrestrial gravity data.";

(3) On the assumption that the altimeter system is stable, and that systematic orbit radial errors for short time periods are constant, the altimeter geoid shows very high frequency details of the geoid or more accurately the sea surface topography. Such high frequency details may also reflect the inexact fulfillments of this assumption or the uncorrected influence of sea state.

(4) Subject to additional data processing corrections which the current state of the SL/2 data precludes, these preliminary results indicate that satellite altimetry will be a valid and useful tool for computing quasi-stationary departures of sea surface topography from the geoid. This practical application is important to oceanographic work related to ocean dynamic phenomena such as circulation patterns, mass water transport, ocean tides, ocean current influences, etc. These in turn relate to air-sea interaction and the knowledge for global numerical weather prediction. Such oceanographic factors also affect our knowledge of pollution dispersion by the oceans, an important guiding factor in waste disposal, and prediction of dispersal and control of oil spill hazards. Further developments on these issues are in Fubara and Mourad [1973];

(5) The preliminary indications are that the general slope of the analytically derived altimeter geoid tends to agree with that computed from purely satellite derived geopotential coefficients and orbit perturbation analysis;

(6) Current orbit computation in which inadequately calibrated altimeter ranges are employed as constraints is not desirable and present no advantage for processing altimeter data to compute the geoid. First, the unmodelled range biases introduce large systematic errors that are not admissible in least squares orbit computation. Such systematic errors cannot be accurately eliminated through modeling unless some valid geodetic controls are used as constraints. Second, the use of orbits computed in this way to deduce a geoid from the same altimeter data with purely differencing or graphical techniques would be misleading. For example, the geoid so deduced would closely match the original geoid used in applying the altimeter ranges as a constraint in

the orbit computation. This type of constraint was involved in the NASA/Wallops orbit but not in the NASA/JSC orbit. However, in theory, other salient features of the NASA/JSC orbit computations are much less sophisticated than that of NASA/Wallops;

(7) Deduction of the geoid from satellite altimetry cannot be achieved by merely subtracting altimeter ranges from the corresponding geodetic heights of the satellite unless (a) the satellite orbit is errorless, (b) the altimeter does not drift, and (c) the altimeter system biases are either non-existent or are absolutely known. Therefore, in practice, at this time, satellite altimetry ranges cannot be regarded as representing direct determination of absolute geoid heights as assumed in Rapp [1971] simulation studies; however, the general conclusions of those studies are still valid with respect to the need for inclusion of terrestrial geodetic data for analytical processing of satellite altimetry data. At this time marine geodesy, involving the use of astrogravimetric and satellite geodesy techniques, appears indispensable for the full achievement of satellite altimetry objectives of GEOS-C, and the NASA-proposed "Earth and Ocean Physics Applications Program".

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TABLE 1. GEODETIC HEIGHT OF SKYLAB AND A PRIORI GEOIDAL HEIGHTS INVOLVED IN DATA ANALYSIS
(All values are in meters)

Skylab Geodetic Heights Based on		A Prior Geoidal Height
NASA/JSC Orbit	NASA/Wallops Orbit	
438752.0	438771.9	-41.0
55.3	75.0	-41.7
56.0	75.6	-41.8
56.7	76.2	-42.0
59.6	79.4	-42.7
63.5	82.7	-43.5
66.7	86.0	-44.3
70.2	89.3	-45.2
70.8	89.8	-45.3
71.3	90.3	-45.5
73.9	93.0	-45.2
76.5	95.4	-46.9
77.0	95.9	-47.0
77.6	96.4	-47.1
80.4	438798.7	-47.8
83.2	438801.6	-48.7
83.8	2.1	-48.8
84.3	2.5	-49.0
86.7	4.7	-49.0
88.0	6.0	-49.1
88.8	7.0	-49.2
89.3	7.5	-49.3
89.7	7.9	-49.3
92.2	10.2	-49.5
438794.9	438812.5	-49.7

TABLE 2. ANALYTICALLY ADJUSTED RANGES BASED ON NASA/JSC ORBIT EREP PASS 9 OF SL-2
(All values in meters)

Measured Altimeter Ranges		Based on NASA/JSC Orbit Adjusted Altimeter Ranges	
NASA/JSC	NASA/Wallops	NASA/JSC	NASA/Wallops
438814.5	438906.8	438703.8	438704.4
18.6	10.3	07.8	07.9
19.2	11.9	08.5	09.5
19.8	12.3	09.1	09.9
23.4	15.6	12.6	13.2
27.7	19.9	16.9	17.5
31.4	22.2	20.7	19.8
35.2	26.7	24.4	24.3
35.6	26.9	24.8	24.5
36.2	27.9	25.5	25.5
38.9	30.6	28.1	28.2
40.8	32.5	30.0	30.1
41.6	33.2	30.8	30.8
42.0	33.9	31.3	31.5
45.6	36.1	34.8	33.7
48.5	39.9	37.8	37.5
49.1	41.3	38.4	38.9
49.4	41.6	38.7	39.2
51.8	43.1	41.1	40.7
53.2	44.7	42.5	42.3
54.3	45.0	43.6	42.6
55.1	45.9	44.3	43.5
54.6	46.6	43.8	44.2
56.8	47.8	46.1	45.4
438859.7	438950.7	438749.0	438738.3
		Geodetic Calibration Constant	
		-110.7	-202.4

TABLE 3. ANALYTICALLY ADJUSTED RANGES BASED ON NASA/WALLOPS ORBIT EREP PASS 9 OF SL-2

Measured Altimeter Ranges in meters		Based on NASA/Wallops Orbit Adjusted Altimeter Ranges in meters	
NASA/JSC	NASA/Wallops	NASA/JSC	NASA/Wallops
438814.5	438906.8	438722.5	438723.2
18.6	10.3	26.6	26.7
19.2	11.9	27.3	28.3
19.8	12.3	27.9	28.7
23.4	15.6	31.4	32.0
27.7	19.9	35.7	36.3
31.4	22.2	39.5	38.6
35.2	26.7	43.2	43.1
35.6	26.9	43.7	43.3
36.2	27.9	44.2	44.3
38.8	30.6	46.8	47.0
40.8	32.5	48.9	48.9
41.6	33.2	49.6	49.6
42.0	33.9	50.0	50.3
45.6	36.1	53.6	52.5
48.5	39.9	56.5	56.3
49.1	41.3	57.2	57.7
49.4	41.6	57.5	58.0
51.8	43.1	59.9	59.5
53.2	44.7	61.2	61.1
54.3	45.0	62.4	61.4
55.0	45.9	63.1	62.3
54.6	46.6	62.6	63.0
56.8	47.8	64.9	64.2
438859.7	438950.7	438767.8	438767.1
		Geodetic Calibration Constant	
		-91.9	-183.6

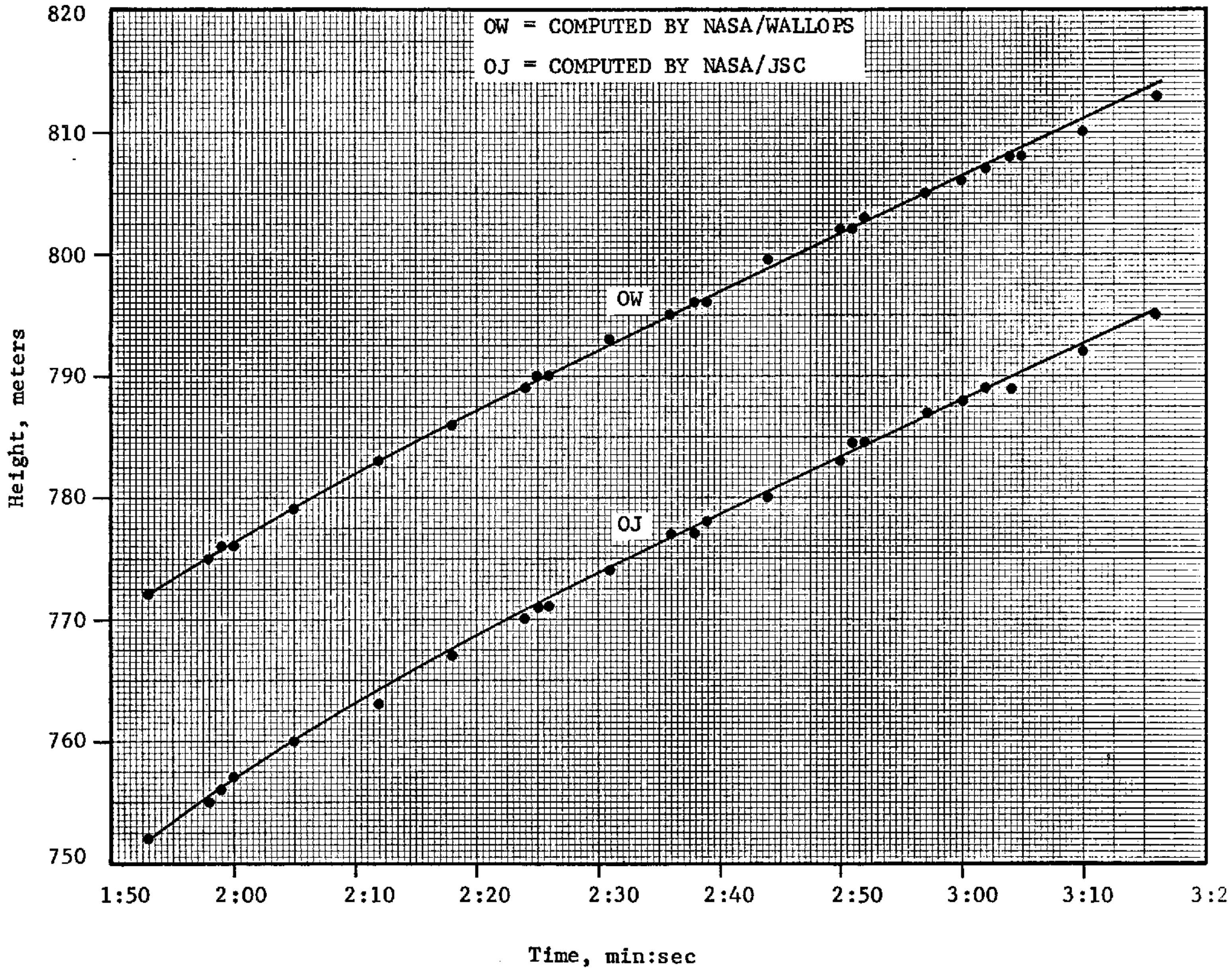

FIGURE 2. GEODETIC HEIGHT OF SKYLAB (SL-2 EREP PASS #9)

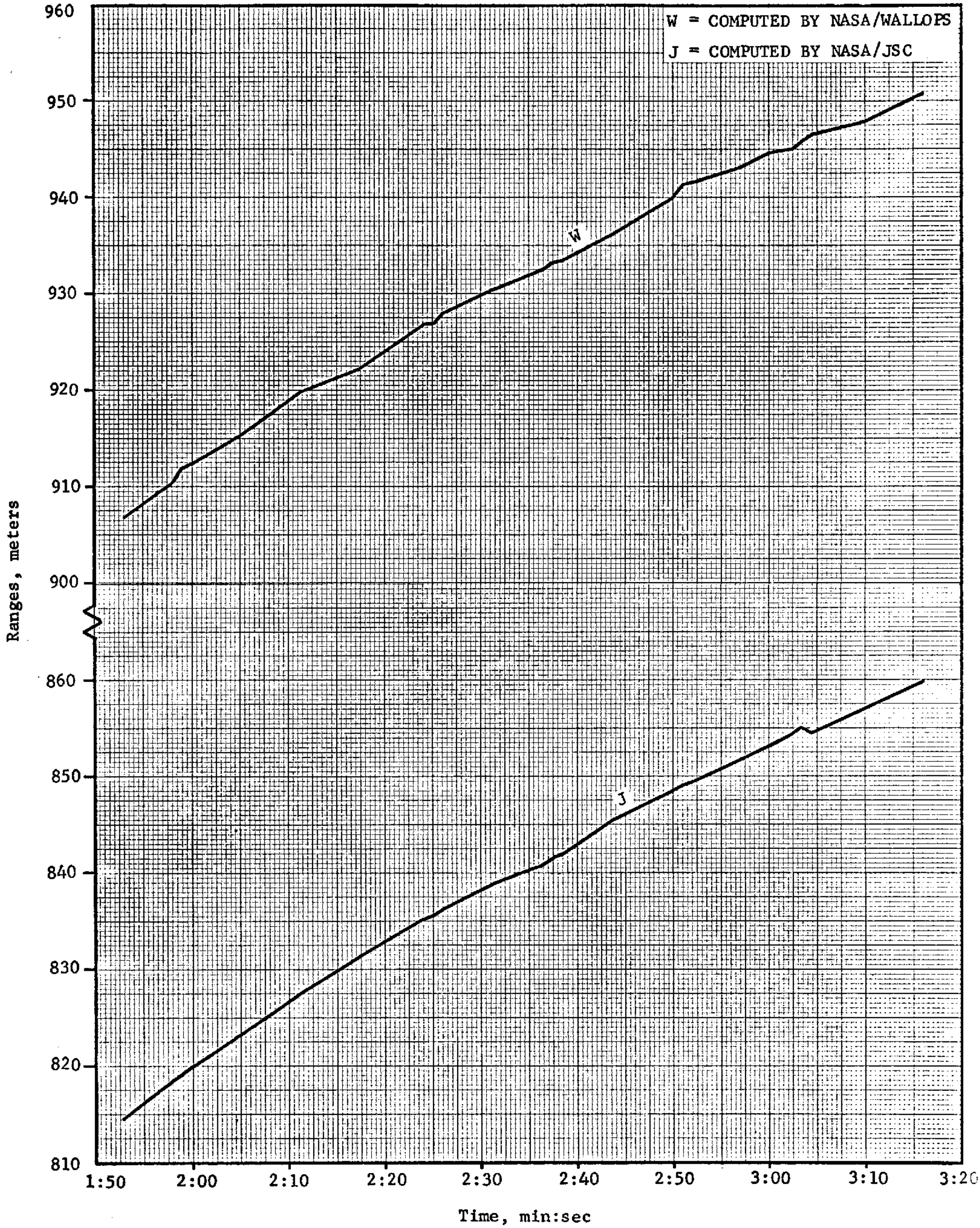

FIGURE 3. ALTIMETER RANGES (SL-2 EREP PASS #9)

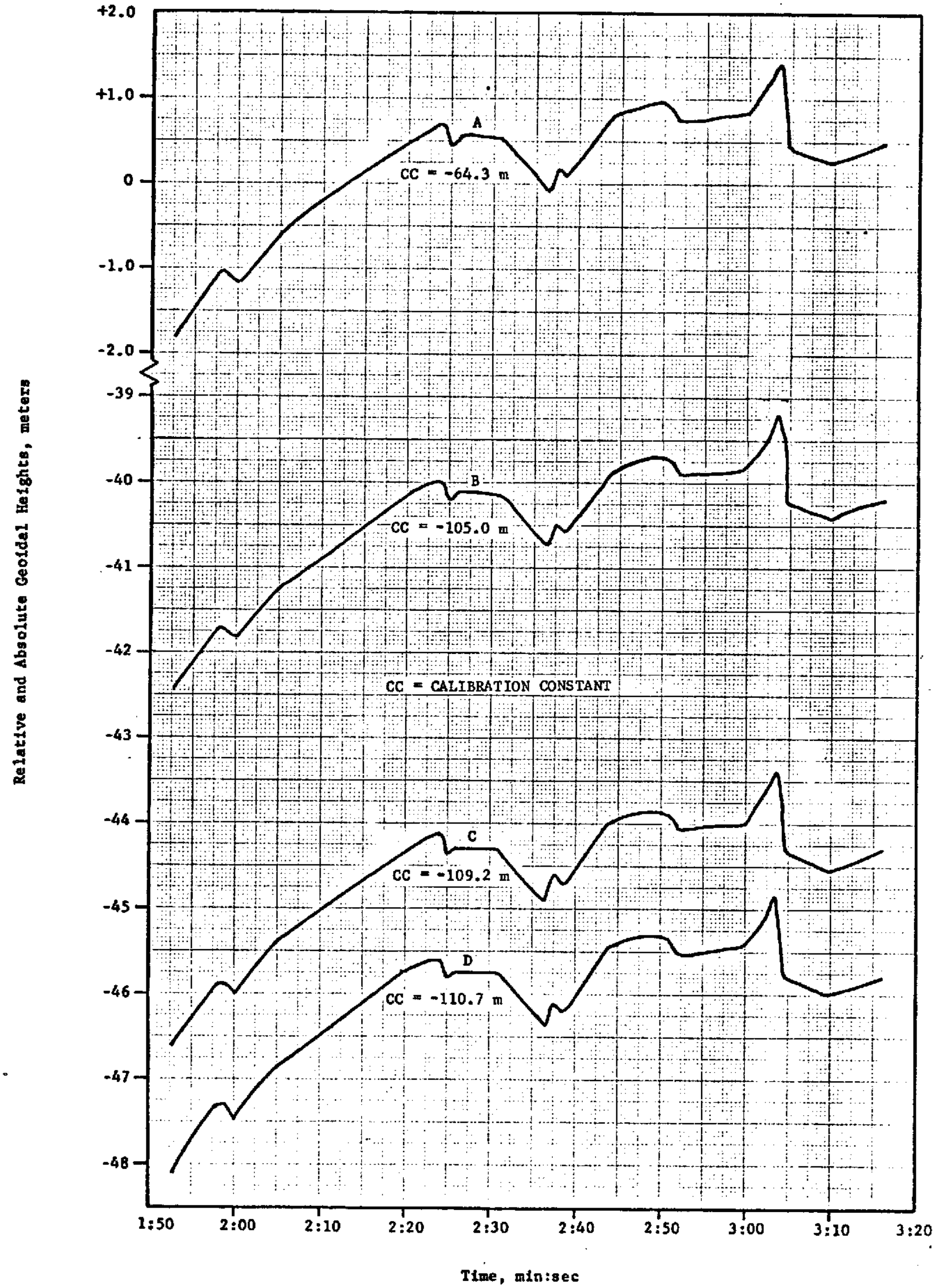

FIGURE 4. DEPENDENCY OF CALIBRATION CONSTANT AND GEOIDAL HEIGHTS GEODETIC CONTROL (GROUND TRUTH)

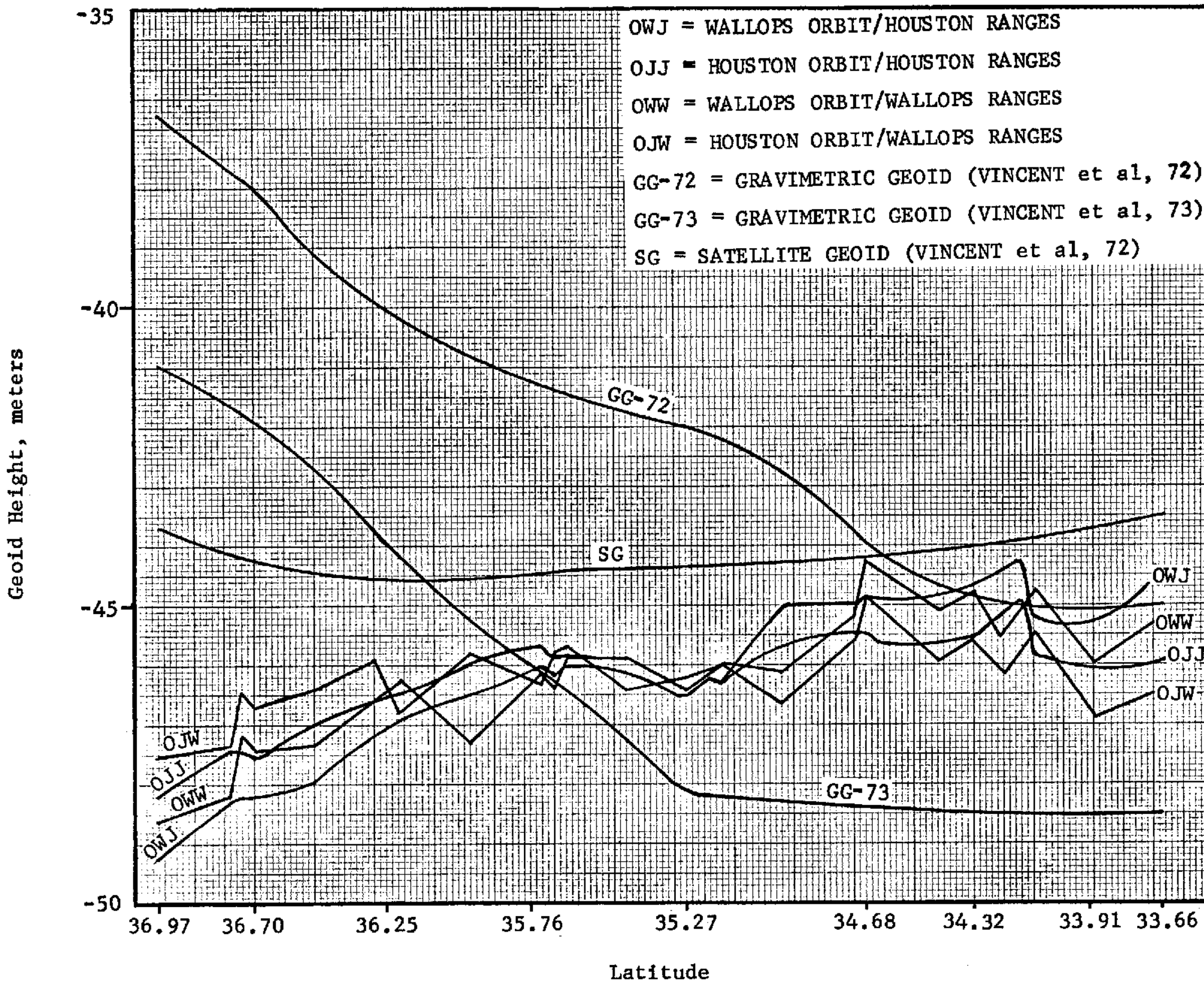

FIGURE 5. CONVENTIONAL GEOID AND SATELLITE ALTIMETRY GEOID SEGMENTS

TABLE 4. EFFECT OF ERRORS IN A PRIORI
GEOID HEIGHT INPUTS
(values in meters)

Control Result		Results from Input Errors			
A priori Geoid Input	Adjusted Geoid Heights	A priori Geoid Input	Adjusted Geoid Heights	A priori Geoid Input	Adjusted Geoid Heights
-41.0	-48.2	-46.0	-35.3	-47.0	-48.3
-41.7	-47.4	-33.7	-34.6	-35.7	-47.5
-41.8	-47.5	-44.8	-34.6	-47.8	-47.5
-42.0	-47.5	-32.0	-34.7	-36.0	-47.6
-42.7	-47.0	-43.7	-34.1	-48.7	-47.0
-43.5	-46.5	-31.5	-33.6	-37.5	-46.6
-44.3	-46.0	-43.3	-33.1	-50.3	-46.1
-45.2	-45.7	-31.2	-32.8	-39.2	-45.8
-45.3	-45.9	-42.3	-33.0	-51.3	-46.0
-45.5	-45.8	-29.5	-33.0	-39.5	-45.9
-46.2	-45.9	-41.2	-32.0	-52.2	-45.9
-46.9	-46.5	-28.9	-33.6	-40.9	-46.5
-47.0	-46.2	-40.0	-33.3	-53.0	-46.3
-47.1	-46.3	-27.1	-33.4	-41.1	-46.3
-47.8	-45.6	-38.8	-32.7	-53.8	-45.6
-48.7	-45.4	-26.7	-32.5	-42.7	-45.5
-48.8	-45.5	-37.8	-32.6	-54.8	-45.5
-49.0	-45.6	-25.0	-32.7	-43.0	-45.7
-49.0	-45.6	-36.0	-32.7	-55.0	-45.6
-49.1	-45.5	-23.1	-32.6	-43.1	-45.6
-49.2	-45.2	-34.2	-32.3	-55.2	-45.3
-49.3	-44.9	-21.3	-32.1	-43.3	-45.0
-49.3	-45.9	-32.3	-33.0	-55.3	-46.0
-49.5	-46.1	-19.5	-33.2	-43.5	-46.2
-49.7	-45.9	-30.7	-33.0	-55.7	-45.9

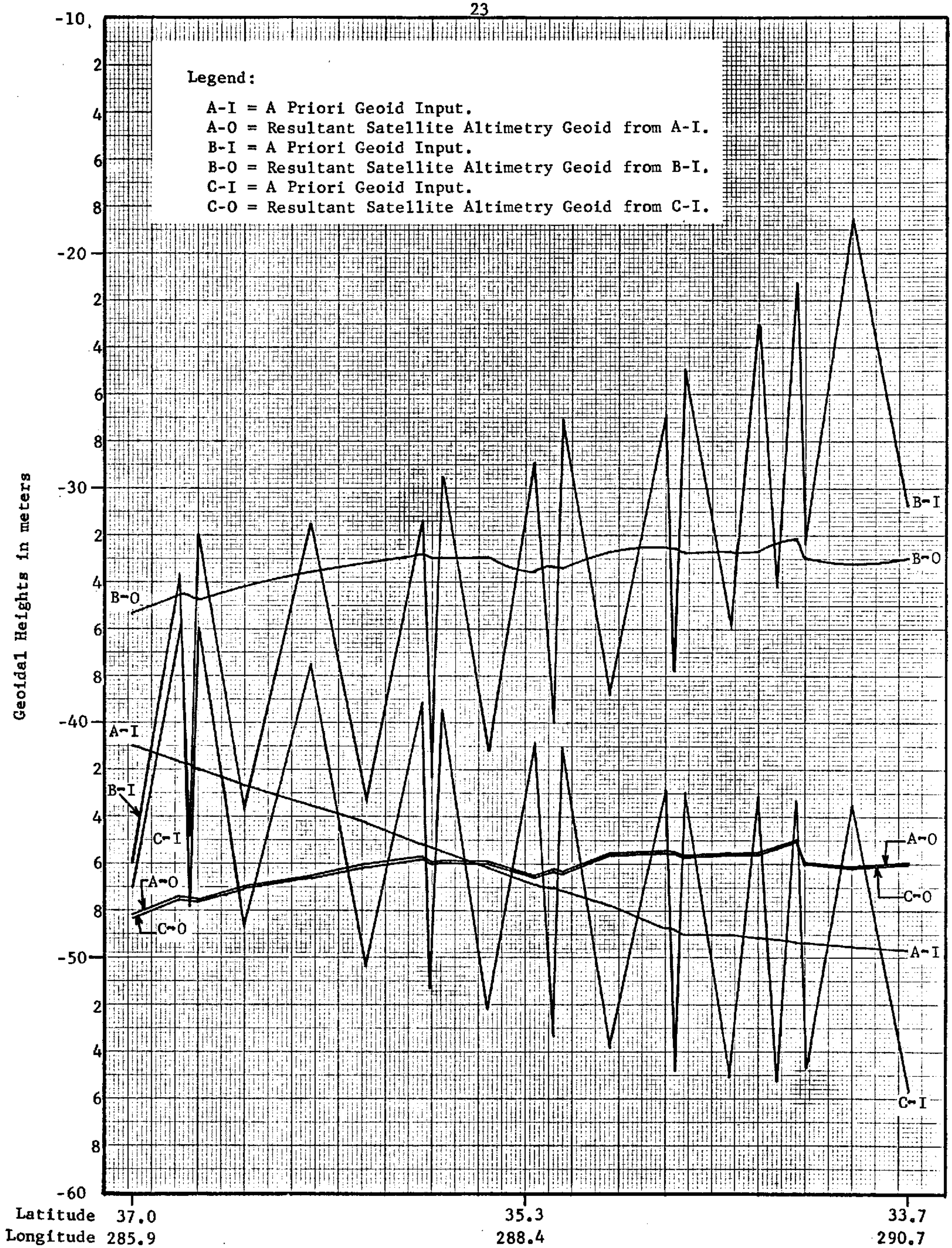

FIGURE 6. EFFECT OF ERRORS IN A PRIORI GEOID HEIGHT INPUTS

TABLE 5. ANALYTICALLY COMPUTED GEOIDAL HEIGHTS FROM DIFFERENT DATA COMBINATIONS (Values in meters)

	OJJ	OJW	OWW	OWJ
	-48.2	-47.5	-48.7	-49.3
	-47.4	-47.4	-48.3	-48.3
	-47.5	-46.5	-47.3	-48.3
	-47.5	-46.7	-47.5	-48.3
	-47.0	-46.4	-47.4	-48.0
	-46.5	-45.9	-46.4	-46.9
	-46.0	-46.7	-47.4	-46.5
	-45.7	-45.8	-46.2	-46.0
	-45.9	-46.3	-46.5	-46.1
	-45.8	-45.8	-46.0	-46.0
	-45.9	-45.7	-46.0	-46.1
	-46.5	-46.4	-46.5	-46.5
	-46.2	-46.2	-46.3	-46.3
	-46.3	-46.1	-46.1	-46.3
	-45.6	-46.7	-46.2	-45.1
	-45.4	-45.7	-45.3	-45.0
	-45.5	-44.9	-44.4	-44.9
	-45.6	-45.1	-44.5	-45.0
	-45.6	-46.0	-45.2	-44.8
	-45.5	-45.7	-44.9	-44.7
	-45.2	-46.2	-45.6	-44.6
	-45.0	-45.8	-45.2	-44.4
	-45.9	-45.3	-44.9	-45.3
	-46.1	-46.8	-46.0	-45.3
	-45.9	-46.6	-45.4	-44.7
Average Std. Error*	± 3.1	± 3.0	± 3.2	± 3.2

* Std. Error = square root of main diagonal element of variance covariance matrix of the least squares adjustment

Key to Data Combination

OJJ = NASA/JSC Orbit and NASA/JSC Altimeter Ranges

OJW = NASA/JSC Orbit and NASA/Wallops Altimeter Ranges

OWW = NASA/Wallops Orbit and NASA/Wallops Altimeter Ranges

OWJ = NASA/Wallops Orbit and NASA/JSC Altimeter Ranges

TABLE 6. APPARENT "GEOIDAL HEIGHTS" FROM GEODETIC HEIGHT OF SKYLAB ORBIT MINUS ALTIMETER RANGE

OJJ	OJW	OWW	OWJ
-62.5	-154.8	-134.9	-42.6
-62.3	-155.0	-135.3	-43.6
-63.2	-155.9	-136.3	-43.6
-63.2	-155.6	-136.1	-43.6
-63.7	-156.0	-136.2	-44.0
-64.2	-156.4	-137.2	-45.0
-64.7	-155.5	-136.2	-45.4
-65.0	-156.5	-137.4	-45.9
-64.8	-156.1	-137.1	-45.8
-64.9	-156.6	-137.6	-45.9
-64.9	-156.7	-137.6	-45.8
-64.3	-156.0	-137.1	-45.4
-64.5	-156.2	-137.3	-45.7
-64.4	-156.3	-137.5	-45.6
-65.1	-155.7	-137.4	-46.9
-65.3	-156.7	-138.3	-46.9
-65.3	-157.5	-139.2	-47.0
-65.1	-157.3	-139.1	-46.9
-65.1	-156.4	-138.4	-47.1
-65.2	-156.7	-138.7	-47.1
-65.5	-156.2	-138.0	-47.3
-65.8	-156.6	-138.4	-47.6
-64.8	-156.8	-138.7	-46.7
-64.6	-155.6	-137.6	-46.6
-64.8	-155.8	-138.2	-47.2

Key to Data Combination

OJJ = NASA/JSC Orbit and NASA/JSC Altimeter Ranges

OJW = NASA/JSC Orbit and NASA/Wallops Altimeter Ranges

OWW = NASA/Wallops Orbit and NASA/Wallops Altimeter Ranges

OWJ = NASA/Wallops Orbit and NASA/JSC Altimeter Ranges

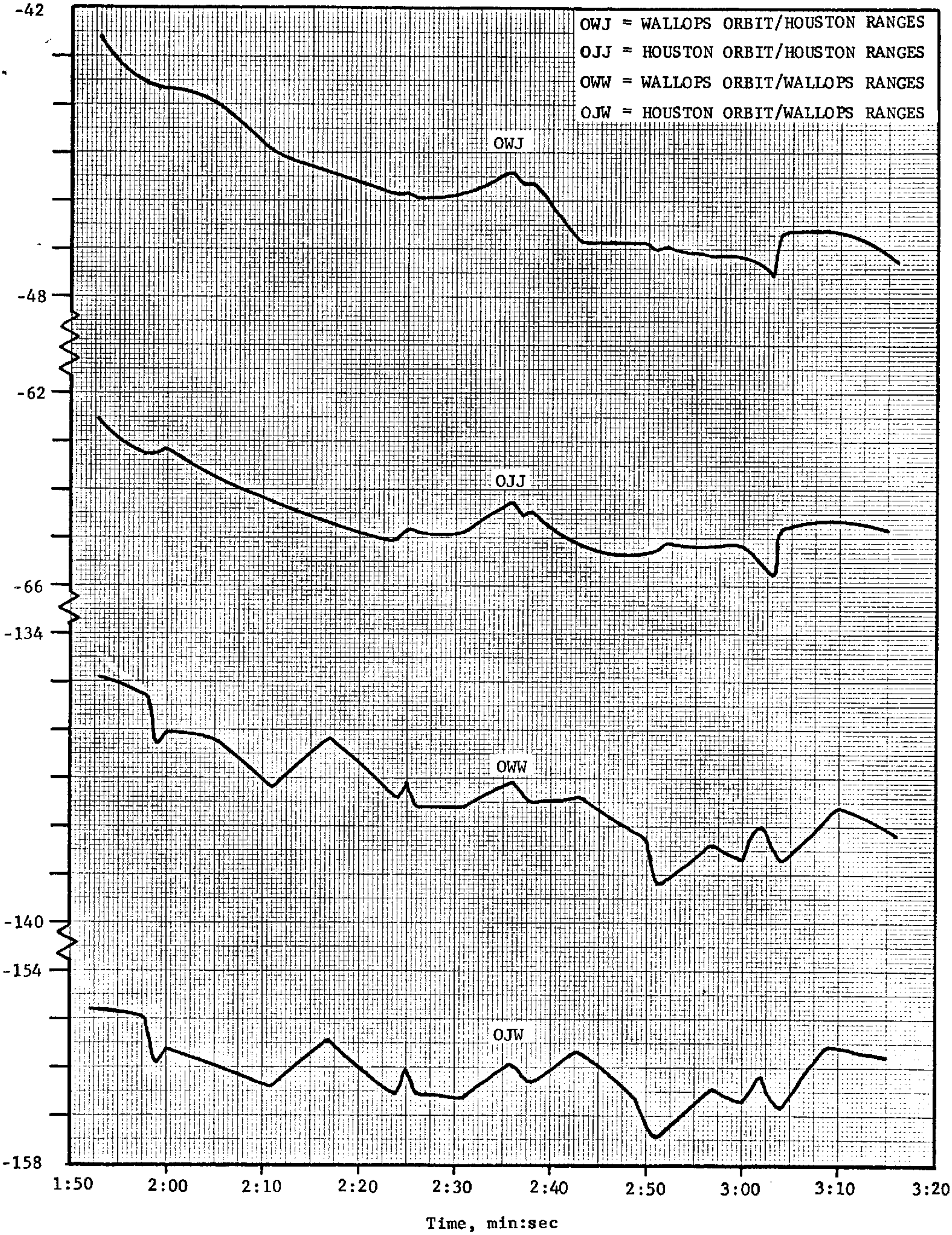

FIGURE 7. SATELLITE HEIGHT MINUS RANGE